3D Magnetization vector inversion based on fuzzy clustering

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04/30/2021

Challenges with magnetic data interpretation

1. Remanence exists in many situations, including resource exploration (e.g., IOCG deposit) and large scale crustal studies
2. Total magnetization directions unknown.
3. Simply ignoring remanence results in misinterpretation of magnetic anomalies.
4. In case of strong remanence, conventional susceptibility inversion (e.g., Li & Oldenburg, 1996) fails.

Current approaches

1. **Direction estimation**
    - Helbig (1962), Lourenco & Morrison (1973), Fedi et al. (1994), Phillips (2005), Dannemiller & Li (2006), etc.
    - Only good for isolated and compact bodies
2. **Transformation**
    - Nabighian (1972), Paine et al., (2001), Li et al. (2010), Pilkington & Beiki (2013), etc.
    - For 3D, only weakly dependent transforms available
    - Only susceptibilities are recovered.
3. **Magnetization vector inversion**
    - Wang et al. (2004), Kubota & Uchiyama (2005), Lelievre & Oldenburg (2009), Ellis et al., (2012), etc.
    - Recovers total magnetization vectors
    - Applicable to sources of any geometry and distribution

Conventional Magnetization vector inversion

$$\varphi(\mathbf{m}) = \left\| \mathbf{W}_d (\mathbf{d}^{obs} - \mathbf{G}\mathbf{m}) \right\|_2^2 + \beta \left\| \mathbf{W}_m (\mathbf{m} - \mathbf{m}_{ref}) \right\|_2^2$$

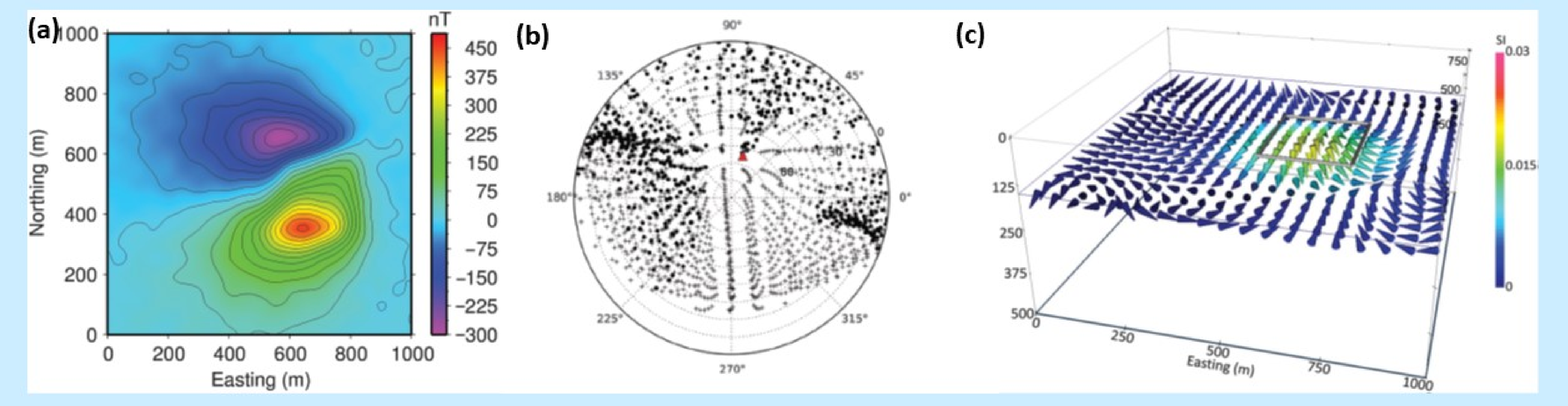

Figure 1: (a) Total-field anomaly caused by a source body magnetized in the direction of (I = 65^0, D = 75^0) under inducing field in the direction of (I = 10^0, D = -25^0). (b) Magnetization directions recovered from unconstrained MVI displayed in a polar plot. (c) The recovered magnetization vectors at z = 150 m. (Li and Sun, 2016)

Magnetization clustering inversion

$$\varphi(\mathbf{m}; u_{jk}, \mathbf{v}_k) = \left\| \mathbf{W}_d (\mathbf{d}^{obs} - \mathbf{G}\mathbf{m}) \right\|_2^2 + \beta \left\| \mathbf{W}_m (\mathbf{m} - \mathbf{m}_{ref}) \right\|_2^2$$

$$+ \lambda \left(\sum_{j=1}^{M} w_j^2 \sum_{k=1}^{C} u_{jk}^q \left\| \hat{\mathbf{J}}_j - \hat{\mathbf{v}}_k \right\|_2^2 + \eta \sum_{k=1}^{C} \left\| \hat{\mathbf{v}}_k - \hat{\mathbf{t}}_k \right\|_2^2 \right)$$

- The clustering term limits recovered magnetization directions to only a few possibilities
- Encourages ***clustering*** among inverted directions in an inclination-declination plane
- Results in ***region-wise consistency*** among inverted directions in spatial domain

What is clustering?

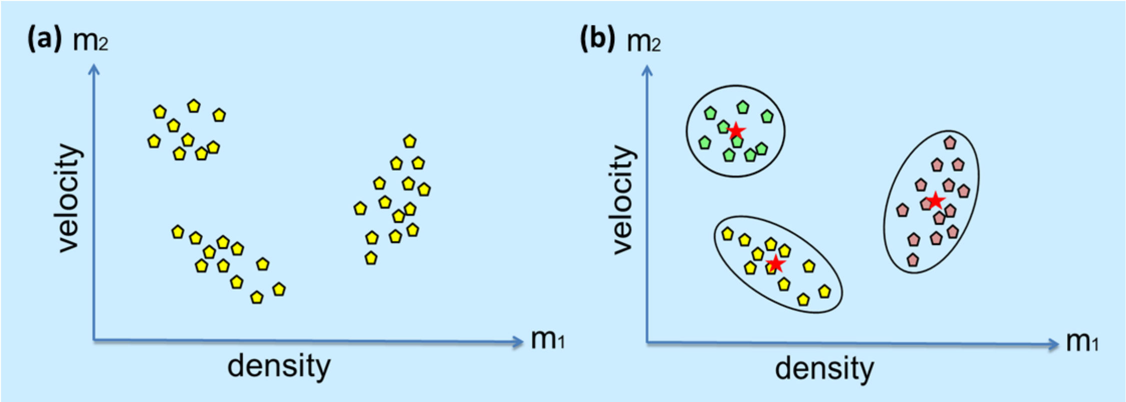

Figure 2: (a) Data to be clustered. (b) Clustering results.

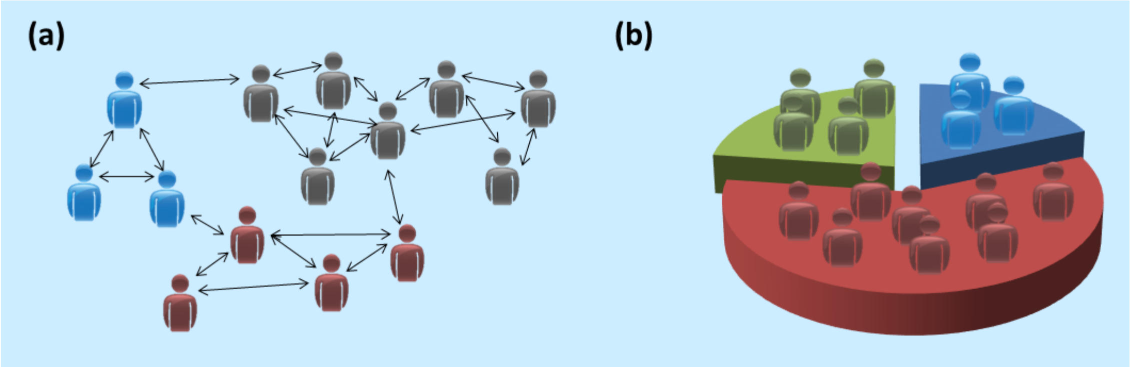

Figure 3: Applications of clustering: (a) Social network analysis. (b) Market segmentation.

Iterative minimization algorithm

Given initial model, $\mathbf{m}$, initial cluster centers, $\mathbf{v}_k$.
Choose number of magnetization directions, C.

Repeat for i = 1, 2, …

1. Update membership value u_{jk}

$$u_{jk} = \left\{\left\|\mathbf{m}_j - w_j\hat{\mathbf{v}}_k\right\|_2^2\right\}^{-\frac{1}{q-1}} \Big/ \sum_{i=1}^{C}\left\{\left\|\mathbf{m}_j - w_j\hat{\mathbf{v}}_i\right\|_2^2\right\}^{-\frac{1}{q-1}}$$

2. Update model $\mathbf{m}$

$$\left(\mathbf{G}^T\mathbf{W}_d^T\mathbf{W}_d\mathbf{G} + \beta\mathbf{W}_m^T\mathbf{W}_m + \lambda\sum_{k=1}^{C}\hat{\mathbf{U}}_k\right)\mathbf{m} = \mathbf{G}^T\mathbf{W}_d^T\mathbf{W}_d\mathbf{d}^{obs} + \beta\mathbf{W}_m^T\mathbf{W}_m\mathbf{m}_{ref} + \lambda\sum_{k=1}^{C}\tilde{\mathbf{U}}_k\tilde{\mathbf{W}}\tilde{\mathbf{V}}_k$$

3. Update cluster center,

$$\hat{\mathbf{v}}_k = \frac{\sum_{j=1}^{M}u_{jk}^q w_j\mathbf{m}_j + \eta\hat{\mathbf{t}}_k}{\sum_{j=1}^{M}u_{jk}^q w_j^2 + \eta}$$

4. Calculate relative model update $\delta = \left\|\mathbf{m}^{(i)} - \mathbf{m}^{(i-1)}\right\|_2 / \left\|\mathbf{m}^{(i-1)}\right\|_2$
5. Calculate data misfit ϕ_d
6. Update spatial weights, w_j , based on latest model

Until $\delta < \tau$ and $\phi_d \approx \phi_d^*$

Application to field data

- Furnas southwest iron oxide-copper-gold (IOCG) deposit in Carajas Mineral Province, Brazil.
- Copper-gold disseminated mineralization resulted from biotite-garnet-grunerite-magnetite hydrothermal alteration (Leao-Santos et al., 2015).
- High-grade chalcopyrite and bornite ores have strong associations with the presence of magnetite.

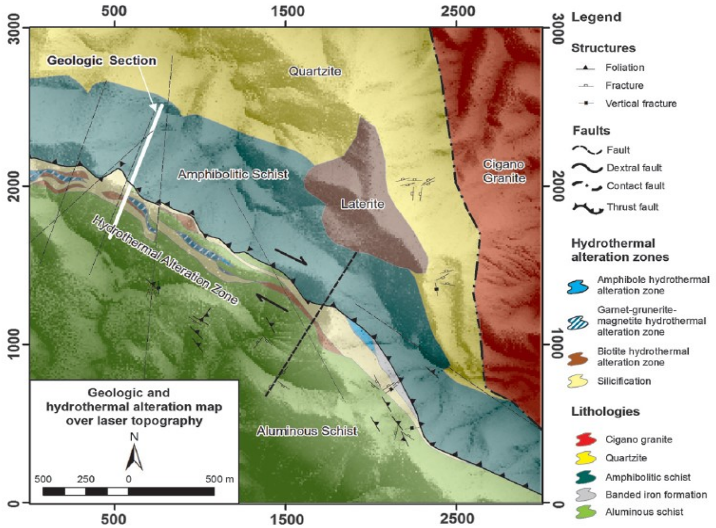

Figure 4: Alteration map in the study area. The mineralization zone is distributed along the northwest-southeast thrust fault (Leao-Santos et al., 2015).

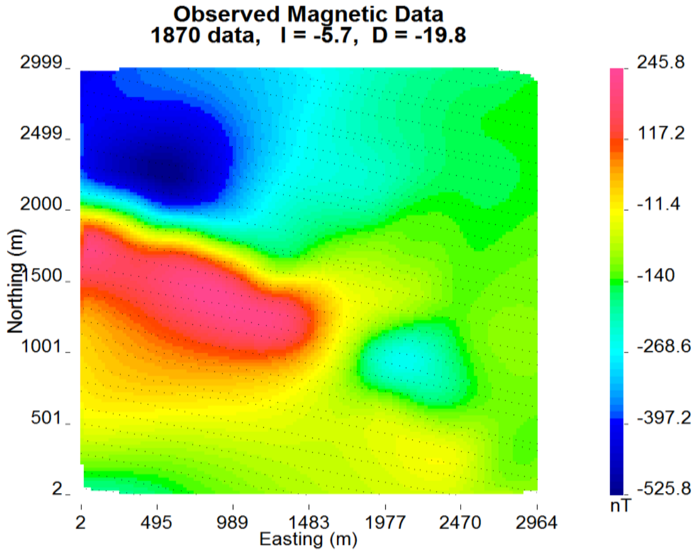

Figure 5: The magnetic total-field anomalies measured over the southwest Furnas deposit from an airborne magnetic survey.

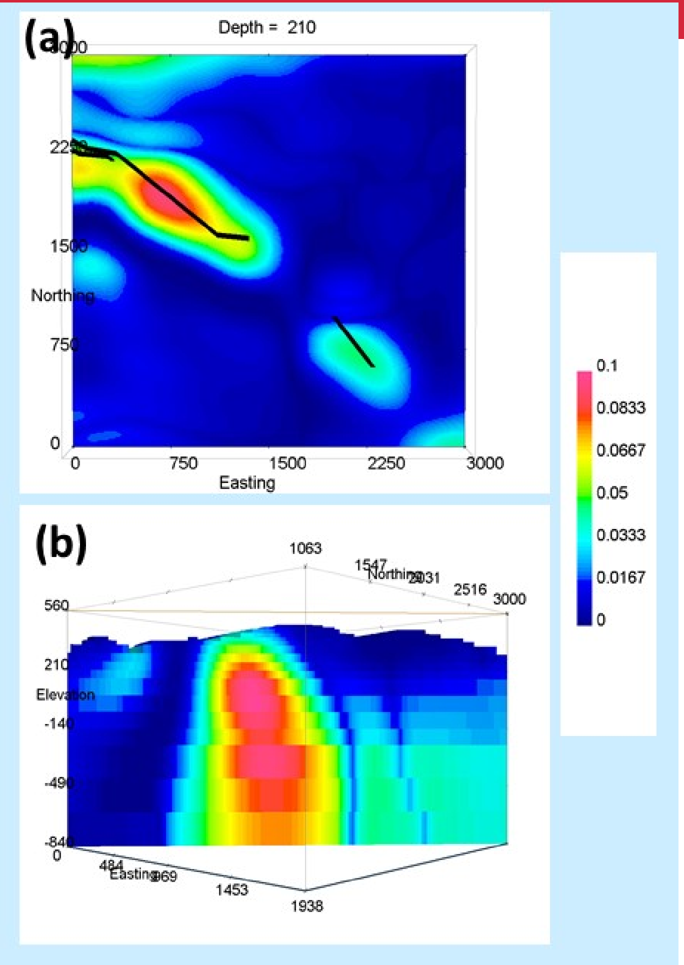

Figure 6: (a) Recovered effective susceptibilities from MCI by assuming 3 clusters. Black lines mark the locations of known orebodies. (b) A cross-section.

Conclusions

- MCI is an effective tool for interpreting magnetic data complicated by remanence at low latitudes.
- Allows to quantify the uncertainty of recovered magnetization directions by adjusting number of clusters.
- We have developed an inversion algorithm for estimating magnetization vectors and a workflow for assessing uncertainty.
- Successfully applied to field data.