Students Course Description for Math 4331-4332

These two courses are perhaps the most important steppingstone in our undergraduate sequence to careers or further study requiring any rigorous mathematics. They provide a basic but careful (theorem-proof style) introduction to convergence and continuity and other topics in analysis that extends these notions beyond the real line to the more general setting needed for many areas of science, engineering, economics, etc., and of course later math courses. This setting includes metric spaces and spaces of functions. It is assumed that the student is familiar with the material of MATH 3333, including basic properties of continuous, differentiable, and Riemann integrable functions on the real line, that they have some facility with epsilon-delta proofs, and that they are able to follow detailed line-by-line logical arguments in undergraduate mathematics.

Possible texts include: (Note that several of these texts begin with chapters devoted to material from prerequisite classes, of course most of this will not be covered in 4331-4332).

- N. L. Carothers, *Real Analysis*, Cambridge (has excellent problem sets)
- K. Davidson and A. P. Donsig, *Real Analysis and Applications: Theory in Practice*. ISBN: 978-1461499008
- W. Rudin, *Principles of Mathematical Analysis*, 3rd edition, ISBN-13: 978-0070542358
- T. Tao, *Analysis II*, 3rd edition, ISBN-13: 978-9380250656

Some instructors may instead distribute lecture notes.

The topics below represent the two semester course—students just taking the first semester may only see two or three of these Sections. Most results are proved in full. Instructors may re-order topics both within and between the semesters, and may also add other topics not listed here (there is a variant of this syllabus aimed at instructors with more detailed suggestions). **All instructors will cover Sections I, II, and V thoroughly, as well as parts of Sections III, IV, and/or VI.**

I. **Normed vector spaces and metric spaces**
   Normed vector spaces. Metrics. Examples: Euclidean space R^n (a key example within each of the following topics), function spaces. Metric space topology: open and closed sets; equivalent norms and metrics; sequences, convergence, and subsequences; interior, closure, boundary; Cauchy sequences, completeness and completion; continuity and limits; compactness including characterization of compactness in metric spaces and the extreme value theorem; uniform continuity. Connectedness, path connectedness, and continuous functions on connected sets. Baire category theorem and applications. Continuous linear maps and dual spaces, finite dimensional case. Brief discussion of inner products and orthonormal sets and expansions. Separable Hilbert spaces (as time permits).

II. **Sequences and series of functions**

Series of numbers and convergence tests. Uniform convergence of sequences and series of functions. Uniform convergence and continuity and limits. Weierstrass M-test. Uniform convergence and derivatives. Uniform convergence and integration. Theory of power series (the main results on functions defined by power series, operations on power series, connection with Taylor series, etc). Elementary functions as power series (exponential, logarithm, trigonometric functions). Polynomial approximation. Convolutions. Weierstrass Approximation Theorem. Stone-Weierstrass Theorem.

III. **A more general integral**

The Lebesgue integral or Riemann-Stieltjes integral, Bounded variation, etc.

IV. **Fourier series**

Trigonometric identities. Orthogonal and orthonormal functions. Fourier series. Best L^2 approximation. Bessel inequality. Parseval's identity. Riemann-Lebesgue lemma. Convergence results for Fourier series (L^2, uniform, pointwise), and applications to physics, these as time permits.

V. **Arzelà-Ascoli theorem and compactness in function spaces**

VI. **Multivariable differential calculus**

The derivative f' as linear map (total derivative). Differentiability and continuity, etc. The contraction mapping theorem and applications. Proof of the inverse and implicit function theorem in $\mathbb{R}^n$. Further topics as time permits.

VII. **Other possible topics**